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Research Paper

A ξ-PROJECTIVELY FLAT CONNECTION ON KENMOTSU MANIFOLDS

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Abstract. In this paper, we introduce a semi-symmetric non-metric connection on η-Kenmotsu manifolds that changes an η-Kenmotsu manifold into an Einstein manifold. Next, we consider an especial version of this connection and show that every Kenmotsu manifold is ξ-projectively flat with respect to this connection. Also, we prove that if the Kenmotsu manifold M is a ξ-concircular flat with respect to the new connection, then M is necessarily of zero scalar curvature. Then, we review the sense of ξ-conformally flat on Kenmotsu manifolds and show that a ξ-conformally flat Kenmotsu manifold with respect to the new connection is an η-Einstein with respect to the Levi-Civita connection. Finally, we prove that there is no ξ-conharmonically flat Kenmotsu manifold with respect to this connection.

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1. Introduction and Background

The sense of Kenmotsu manifolds was introduced for the first time in [4] by K. Kenmotsu. He proved that a locally Kenmotsu manifold is a warped product $I \times_f N$ of an interval I and a Kaehler manifold N with warping function $f(t) = se^t$, where s is a non-zero constant. The semi-symmetric connections was first defined by Friedman and Schouten ([5], [1]). The linear connection $\overline{\nabla}$ is named a semi-symmetric connection if the following relation holds:

$$\overline{T}(X,Y) = u(Y)X - u(X)Y, \tag{1.1}$$

where $\overline{T}$ is the torsion tensor of $\overline{\nabla}$ and u is a 1-form. The semi-symmetric connections play a prominent role in Riemannian geometry. In [7], Yano showed that the existence of a semi-symmetric connection with zero curvature tensor is equivalent to being a conformally flat manifold. The Riemannian manifold (M^n, g) is said to be conformally flat if at any point $p \in M$ there is a neighbourhood U around p and a smooth function f on U such that $(U, e^{2f}g)$ is flat. It is well known that Riemannian manifolds with constant sectional curvature are conformally flat. An important tool to being conformally flat is the Weyl conformal curvature tensor that is given by [8]:

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$$C'(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\} + \frac{r}{(n-1)(n-2)}\{g(Y,Z)X - g(X,Z)Y\}, \tag{1.2}$$

where R, S, and Q are respectively the curvature tensor, the Ricci tensor, and the Ricci operator and r denotes to the scalar curvature of n-dimensional manifold M. Hence, a Riemannian manifold is conformally flat if and only if its Weyl conformal curvature tensor vanishes [8]. Furthermore, the Riemannian manifold (M^n, g) is called locally projectively flat manifold if there is a coordinate system (x, U) at every point p such that x maps any geodesic of M to a straight line in R^n. By the Beltrami's theorem being a locally projectively flat Riemannian manifold is equivalent to being of constant sectional curvature. In [6], Soos defined the projective curvature tensor as follow:

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}\{S(Y,Z)X - S(X,Z)Y\}. \tag{1.3}$$

He proved that a Riemannian manifold is locally projectively flat manifold if and only if its projective curvature tensor is identically vanishes. A concircular transformation on Riemannian manifold (M^n, g) is a transformation that maps every geodesic circle of M to a geodesic circle. The concircular curvature tensor was first defined by Yano [9] as follows:

$$Z(X,Y)W = R(X,Y)W - \frac{r}{n(n-1)}\{g(Y,W)X - g(X,W)Y\}. \tag{1.4}$$

He showed that a Riemannian manifold which admits a concircular transformation is necessarily of constant scalar curvature [9]. Indeed, the conharmonic curvature tensor for the Riemannian manifold (M^n, g) defines as follows [3]:

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\}. \tag{1.5}$$

A Riemannian manifold with zero conharmonic curvature tensor is called a conharmonically flat manifold. Semi-symmetric and quarter-symmetric connections on Kenmotsu manifolds have been studied by many authors. Recently, Haseeb and Prasad defined a semi-symmetric connection on Kenmotsu manifolds and proved that an n-dimensional conharmonically flat η-Einstein Kenmotsu manifold with respect to the semi-symmetric connection is of quasi-constant curvature and has zero scalar curvature [2]. The almost contact metric manifold $(M, \varphi, \xi, \eta, g)$ is called η-Einstein manifold if its Ricci tensor satisfies:

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y), \tag{1.6}$$

where a and b are smooth functions on M. Similar to (1.1), the linear connection $\overline{\nabla}$ on smooth manifold M is named quarter-symmetric if

$$\overline{T}(X,Y) = u(Y)\phi(X) - u(X)\phi(Y), \tag{1.7}$$

where u is a 1-form, ϕ is a $(1,1)$ tensor field, and $\overline{T}$ is the torsion tensor of $\overline{\nabla}$. Thereafter, Zhao and et al. defined a quarter-symmetric connection on Kenmotsu manifolds and proved that every ξ-conformally flat Kenmotsu manifold with respect to the quarter-symmetric connection is an η-Einstein. They also showed that an n-dimensional ξ-concirculary Kenmotsu manifold with respect to the quarter-symmetric is of constant scalar curvature $r = -n(n-1)$

[10] (for more see the references inside [2] and [10]).
Motivation by these works, we define a new semi-symmetric connection on Kenmotsu manifolds and get some interesting results. The paper is organised as follows: In section 2, we give some definitions and facts on Kenmotsu manifolds. In section 3, we define a new semi-symmetric connection on Kenmotsu manifolds that changes an η-Einstein Kenmotsu manifold to an Einstein manifold. In section 4, we consider an especial version of this connection and prove that if the Kenmotsu manifold M is a ξ-concircular flat with respect to the new connection, then M is necessarily of zero scalar curvature. Then, we review the sense of ξ-conformally flat on Kenmotsu manifolds and show that a ξ-conformally flat Kenmotsu manifold with respect to the new connection is an η-Einstein with respect to the Levi-Civita connection. Finally, we prove that there is no ξ-conharmonically flat Kenmotsu manifold with respect to this connection.

2. **Preliminaries**

An almost contact metric manifold is a $(2m+1)$-dimensional smooth manifold M with a $(1,1)$ tensor field φ, a vector field ξ, a 1-form η, and a Riemannian metric g satisfying:

$$\varphi^2 = -I + \eta \otimes \xi, \qquad \eta(\xi) = 1, \qquad \varphi(\xi) = 0, \qquad \eta o \varphi = 0, \tag{2.1}$$

$$g(X,Y) = g(\varphi(X), \varphi(Y)) + \eta(X)\eta(Y), \tag{2.2}$$

$$g(\varphi(X), Y) = -g(X, \varphi(Y)), \qquad g(\xi, X) = \eta(X), \tag{2.3}$$

for all $X, Y \in \chi(M)$, where $\chi(M)$ is the set of all smooth vector fields on M. The almost contact metric manifold $(M, \varphi, \xi, \eta, g)$ is called a Kenmotsu manifold if it satisfies:

$$(\nabla_X \varphi)Y = -\eta(Y)\varphi(X) + g(\varphi(X), Y)\xi, \tag{2.4}$$

where ∇ is the Levi-Civita connection of g. In n-dimensional Kenmotsu manifolds M the following relations hold [4]:

$$\nabla_X \xi = X - \eta(X)\xi. \tag{2.5}$$

$$(\nabla_X \eta)(Y) = g(X,Y) - \eta(X)\eta(Y). \tag{2.6}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X. \tag{2.7}$$

$$R(\xi, X)Y = \eta(Y)X - g(X,Y)\xi. \tag{2.8}$$

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X). \tag{2.9}$$

$$S(X,\xi) = -(n-1)\eta(X). \tag{2.10}$$

$$Q(\xi) = -(n-1)\xi. \tag{2.11}$$

Also, in η-Einstein Kenmotsu manifolds, we have [4]:

$$a + b = -(n-1), \qquad X(b) + 2b\eta(X) = 0. \tag{2.12}$$

The linear connection $\overline{\nabla}$ on Riemannian manifold (M,g) is called a metric connection if $\overline{\nabla} g = 0$, otherwise it is called non-metric.

3. A new connection on η-Einstein Kenmotsu manifolds

Let (M,φ,ξ,η,g) be an η-Einstein Kenmotsu manifold whose Ricci tensor is defined by (1.6). We define the semi-symmetric non-metric connection $\overline{\nabla}$ as follows:

$$\overline{\nabla}_X Y = \nabla_X Y - a\eta(X)Y - \frac{b}{n}\eta(X)\eta(Y)\xi, \tag{3.1}$$

where ∇ is the Levi-Civita connection of g. Consider the torsion tensor of $\overline{\nabla}$ as:

$$\overline{T}(X,Y) = \overline{\nabla}_X Y - \overline{\nabla}_Y X - [X,Y], \tag{3.2}$$

we get

$$\overline{T}(X,Y) = a(\eta(Y)X - \eta(X)Y). \tag{3.3}$$

$$(\overline{\nabla}_X g)(Y,Z) = 2\eta(X)\{ag(Y,Z) + \frac{b}{n}\eta(Z)\eta(Y)\}, \tag{3.4}$$

that verify $\overline{\nabla}$ is a semi-symmetric non-metric connection. Now, we have the following theorem.

Theorem 3.1. *Let (M,φ,ξ,η,g) be an η-Einstein Kenmotsu manifold whose Ricci tensor is given by*

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y), \tag{3.5}$$

then M is an Einstein manifold with respect to $\overline{\nabla}$.

Proof. Let $\overline{R}$ be the curvature tensor with respect to $\overline{\nabla}$ which is given by:

$$\overline{R}(X,Y)Z = \overline{\nabla}_X\overline{\nabla}_Y Z - \overline{\nabla}_Y\overline{\nabla}_X Z - \overline{\nabla}_{[X,Y]}Z. \tag{3.6}$$

By a straight calculation, we see

$$\begin{aligned}\overline{\nabla}_X\overline{\nabla}_Y Z = {} & \nabla_X\nabla_Y Z - a\eta(X)\nabla_Y Z - \frac{b}{n}\eta(X)\eta(\nabla_Y Z)\xi - X(a)\eta(Y)Z \\ & - aX(\eta(Y))Z - a\eta(Y)\nabla_X Z + a^2\eta(Y)\eta(X)Z + \frac{ab}{n}\eta(Y)\eta(X)\eta(Z)\xi \\ & - X(\frac{b}{n})\eta(Y)\eta(Z)\xi - \frac{b}{n}X(\eta(Y)\eta(Z))\xi - \frac{b}{n}\eta(Y)\eta(Z)\nabla_X\xi \\ & - (\frac{b}{n} + a)\eta(Y)\eta(Z)\eta(X)\xi.\end{aligned} \tag{3.7}$$

Interchanging X and Y, we obtain

$$\begin{aligned}\overline{R}(X,Y)Z = {} & R(X,Y)Z + \{Y(a)\eta(X) - X(a)\eta(Y)\}Z - \{X(\frac{b}{n})\eta(Y) \\ & - Y(\frac{b}{n})\eta(X)\}\eta(Z)\xi - \frac{b}{n}\{\eta(Y)X - \eta(X)Y\}\eta(Z) \\ & - \frac{b}{n}\{g(Z,X)\eta(Y)\xi - g(Z,Y)\eta(X)\xi\}.\end{aligned} \tag{3.8}$$

Taking a contraction of the above equation yields

$$\begin{aligned}\overline{S}(Y,Z) = {} & S(Y,Z) + \{Y(a)\eta(Z) - Z(a)\eta(Y)\} - \{\xi(\frac{b}{n})\eta(Y) \\ & - Y(\frac{b}{n})\}\eta(Z) - b\eta(Y)\eta(Z) + \frac{b}{n}g(Y,Z).\end{aligned} \tag{3.9}$$

Let us consider the smooth vector fields Y and Z as follows:

$$Y = \eta(Y)\xi + \overline{Y}, \qquad Z = \eta(Z)\xi + \overline{Z}, \tag{3.10}$$

where $\overline{Y}, \overline{Z} \in \ker(\eta)$. By using (2.12) and the above relations, we find

$$Y(a)\eta(Z) - Z(a)\eta(Y) = \xi(\frac{b}{n})\eta(Y) - Y(\frac{b}{n}) = 0. \tag{3.11}$$

Hence, $\overline{S}(Y, Z)$ can be written as:

$$\overline{S}(Y, Z) = S(Y, Z) - b\eta(Y)\eta(Z) + \frac{b}{n}g(Y, Z). \tag{3.12}$$

From (1.6), we get

$$\overline{S}(Y, Z) = (a + \frac{b}{n})g(Y, Z), \tag{3.13}$$

and this completes the proof. □

4. A ξ-projectively flat connection

In this section, we study the notions of ξ-projectively flat, ξ-conharmonically flat, ξ-concircular flat, and ξ-conformally flat on Kenmotsu manifods. Putting $a = 1$ and $b = -n$ in (3.1), we get an especial version of $\overline{\nabla}$ (which we denote it again by $\overline{\nabla}$) that is

$$\overline{\nabla}_X Y = \nabla_X Y - \eta(X)Y + \eta(X)\eta(Y)\xi. \tag{4.1}$$

By the above assumptions, we have

$$\begin{aligned} \overline{R}(X, Y)Z = R(X, Y)Z &+ \{g(Z, X)\eta(Y) - g(Z, Y)\eta(X)\}\xi \\ &+ \{\eta(Y)X - \eta(X)Y\}\eta(Z), \end{aligned} \tag{4.2}$$

and the Ricci tensor becomes

$$\overline{S}(X, Y) = S(X, Y) - g(X, Y) + n\eta(X)\eta(Y). \tag{4.3}$$

Definition 4.1. [10] Let $(M^n, \varphi, \xi, \eta, g)$ be a Kenmotsu manifold, then M is called a ξ-projectively flat manifold with respect to $\overline{\nabla}$ if

$$\overline{P}(X, Y)\xi = 0, \tag{4.4}$$

for all $X, Y \in \chi(M)$, where $\overline{P}(X, Y)Z$ is defined by

$$\overline{P}(X, Y)Z = \overline{R}(X, Y)Z - \frac{1}{n-1}\{\overline{S}(Y, Z)X - \overline{S}(X, Z)Y\}. \tag{4.5}$$

Theorem 4.2. *Let $(M^n, \varphi, \xi, \eta, g)$ be a Kenmotsu manifold, then M is a ξ-projectively flat manifold with respect to the connection $\overline{\nabla}$.*

Proof. Substituting $Z = \xi$ in (4.8), we see

$$\overline{R}(X, Y)\xi = R(X, Y)\xi + \{\eta(Y)X - \eta(X)Y\}, \tag{4.6}$$

using $R(X, Y)\xi = \eta(X)Y - \eta(Y)X$, we conclude that

$$\overline{R}(X, Y)\xi = 0. \tag{4.7}$$

Also, From (4.3) and $S(X, \xi) = -(n-1)\eta(X)$ we obtain

$$\overline{S}(X, \xi) = 0, \tag{4.8}$$

which proves the theorem. □

Definition 4.3. [10] The Kenmotsu manifold $(M^n, \varphi, \xi, \eta, g)$ is called a ξ-concircular flat manifold with respect to the semi-symmetric connection $\overline{\nabla}$, if

$$\overline{Z}(X,Y)\xi = 0, \tag{4.9}$$

for all $X, Y \in \chi(M)$, where $\overline{Z}(X,Y)W$ defines by

$$\overline{Z}(X,Y)W = \overline{R}(X,Y)W - \frac{\overline{r}}{n(n-1)}\{g(Y,W)X - g(X,W)Y\}. \tag{4.10}$$

Theorem 4.4. *If $(M^n, \varphi, \xi, \eta, g)$ is a ξ-concircular flat Kenmotsu manifold with respect to the semi-symmetric connection $\overline{\nabla}$, then M is of zero scalar curvature.*

Proof. Taking a contraction of (4.3), yields $\overline{r} = r$. Let $\{e_1, e_2, ..., e_{2m}, \xi\}$ be an orthonormal basis for $(2m+1)$-dimensional M $(n = 2m+1)$. Since $\overline{R}(X,Y)\xi = 0$, we get

$$\overline{Z}(X,Y)\xi = -\frac{r}{n(n-1)}\{\eta(Y)X - \eta(X)Y\}. \tag{4.11}$$

Substituting $X = \xi$ and $Y = e_i$ to see

$$\eta(Y)X - \eta(X)Y \neq 0, \tag{4.12}$$

at any point $p \in M$. So, if $\overline{Z}(X,Y)\xi = 0$, then M is necessarily of zero scalar curvature. □

ξ-conformally flat manifolds was first introduced by Zhen and et al. [11]. The Kenmotsu manifold $(M^n, \varphi, \xi, \eta, g)$ is said a ξ-conformally flat Kenmotsu manifolds if $\overline{C'}(X,Y)\xi = 0$, where $\overline{C'}(X,Y)Z$ is given by

$$\begin{aligned} \overline{C}'(X,Y)Z = \overline{R}(X,Y)Z &- \frac{1}{n-2}\{\overline{S}(Y,Z)X - \overline{S}(X,Z)Y + g(Y,Z)\overline{Q}X \\ &- g(X,Z)\overline{Q}Y\} + \frac{\overline{r}}{(n-1)(n-2)}\{g(Y,Z)X - g(X,Z)Y\}. \end{aligned} \tag{4.13}$$

This leads us to the following theorem.

Theorem 4.5. *Let $(M^n, \varphi, \xi, \eta, g)$ be a ξ-conformally flat Kenmotsu manifold with respect to the semi-symmetric connection $\overline{\nabla}$, then M is an η-Einstein manifold with respect to the Levi-Civita connection.*

Proof. The equation (4.3) implies that

$$\overline{Q}(X) = Q(X) - X + n\eta(X)\xi. \tag{4.14}$$

Using (4.7), (4.8), and the above relation, we obtain

$$\begin{aligned} \overline{C'}(X,Y)\xi &= \frac{-1}{n-2}\{\eta(Y)Q(X) - \eta(Y)X - \eta(X)Q(Y) + \eta(X)Y\} \\ &+ \frac{r}{(n-1)(n-2)}\{\eta(Y)X - \eta(X)Y\} \\ &= \frac{1}{n-2}\{\eta(Y)\left[-Q(X) + X + \frac{r}{n-1}X\right] \\ &+ \eta(X)\left[Q(Y) - Y - \frac{r}{n-1}Y\right]\}, \end{aligned} \tag{4.15}$$

substituting $Y=\xi$ and $X=e_i$, we get

$$\overline{C'}(e_i,\xi)\xi=\frac{1}{n-2}\left\{-Q(e_i)+e_i+\frac{r}{n-1}e_i\right\}, \tag{4.16}$$

where $\{e_1,e_2,...,e_{2m},\xi\}$ is an orthonormal basis on M. Thus, if $\overline{C'}(e_i,\xi)\xi=0$, then we have

$$Q(e_i)=(\frac{r}{n-1}+1)e_i. \tag{4.17}$$

From (2.10), we find

$$S(X,Y)=(\frac{r}{n-1}+1)g(X,Y)-(\frac{r}{n-1}+n)\eta(X)\eta(Y), \tag{4.18}$$

□

Similarly to ξ-conformally flat manifolds, a Kenmotsu manifold is called a ξ-conharmonically flat manifold if $\overline{C}(X,Y)\xi=0$ where $\overline{C}(X,Y)Z$ defines by

$$\begin{aligned}\overline{C}(X,Y)Z=\overline{R}(X,Y)Z-\frac{1}{n-2}\{&\overline{S}(Y,Z)X-\overline{S}(X,Z)Y\\ &+g(Y,Z)\overline{Q}X-g(X,Z)\overline{Q}Y\}.\end{aligned} \tag{4.19}$$

Thus, we can state the following theorem.

Theorem 4.6. *There is no ξ-conharminically flat Kenmotsu manifold with respect to the semi-symmetric non-metric connection $\overline{\nabla}$.*

Proof. Using (4.7) and (4.8) and putting $Z=\xi$ in the above equation, we conclude that

$$\overline{C}(X,Y)\xi=-\frac{1}{n-2}\{\eta(Y)\overline{Q}(X)-\eta(X)\overline{Q}(Y)\}. \tag{4.20}$$

From (4.14) and by using the orthonormal basis $\{e_1,e_2,...,e_{2m},\xi\}$, we see

$$\overline{C}(e_i,\xi)\xi=-\frac{1}{n-2}\{Q(e_i)-e_i\}. \tag{4.21}$$

Therefore, the equation $\overline{C}(e_i,\xi)\xi=0$ implies that $Q(e_i)=e_i$ and this yields

$$S(X,Y)=g(X,Y)-n\eta(X)\eta(Y), \tag{4.22}$$

and this is impossible because of (2.12). □

Example 4.7. Let $M^7=\{(x_1,x_2,...,x_6,z)\in\mathbb{R}^7; z>0\}$. Putting $\eta=dz$ and let $\{e_1,...,e_7\}$ be an orthonormal basis which is given by

$$e_7=\xi:=\frac{\partial}{\partial z},\qquad e_i=e^{-z}\frac{\partial}{\partial x_i},\qquad i=1,...,6. \tag{4.23}$$

Next, suppose that the $(1,1)$ tensor φ is given by:

$$\varphi(\xi)=0,\qquad \varphi(e_i)=e_{i+3},\qquad \varphi(e_j)=-e_{j-3}, \tag{4.24}$$

where $i=1,2,3$ and $j=4,5,6$. Then (M^7,φ,ξ,η,g) is a Kenmotsu manifold which the Riemannian metric g is defined by [2]:

$$g=e^{2z}\sum_{i=1}^{6}dx^i\otimes dx^i+dz\otimes dz. \tag{4.25}$$

Also, the curvature tensor and the Ricci tensor of M^7 can be written as follows [2]:

$$R(X,Y)Z = -\{g(Y,Z)X - g(X,Z)Y\}, \tag{4.26}$$

$$S(X,Y) = -6g(X,Y). \tag{4.27}$$

From the above equations, we get

$$\begin{aligned}\overline{R}(X,Y)Z = &-\{g(Y,Z)X - g(X,Z)Y\} + \{g(Z,X)\eta(Y) \\ &- g(Z,Y)\eta(X)\}\xi + \{\eta(Y)X - \eta(X)Y\}\eta(Z).\end{aligned} \tag{4.28}$$

$$\overline{S}(X,Y) = -7g(X,Y) + 7\eta(X)\eta(Y). \tag{4.29}$$

5. **Conclusion**

In this paper, we defined a new semi-symmetric non-metric connection on η-Kenmotsu manifolds that changes an η-Kenmotsu manifold into an Einstein manifold. Next, we proved that for $a = 1$ and $b = -n$ every Kenmotsu manifold is ξ-projectively flat with respect to this connection. Also, we showed that if the Kenmotsu manifold M is a ξ-concircular flat with respect to the new connection, then M is necessarily of zero scalar curvature. Thereafter, we demonstrated a ξ-conformally flat Kenmotsu manifold with respect to the new connection is an η-Einstein with respect to the Levi-Civita connection. Finally, we proved that there is no ξ-conharmonically flat Kenmotsu manifold with respect to this connection.

References

[1] A. Friedmann, and J. A. Schouten, Uber die Geometrie der halbsymmetrischen Ubertragungen. *Mathematische Zeitschrift*, **21**(1): 211–223, 1924.

[2] A. Haseeb, and R. Prasad, (2017). Certain curvature conditions in Kenmotsu manifolds with respect to the semi-symmetric metric connection. *Communications of the Korean Mathematical Society*, **32**(4): 1033–1045, 2017.

[3] Y. Ishii, On conharmonic transformations, *Tensor (N. S.)*, **7**: 73–80, 1957.

[4] K. Kenmotsu, A class of almost contact Riemannian manifolds. *Tohoku Mathematical Journal, Second Series*, **24**(1): 93–103, 1972.

[5] J. A. Schouten, Ricci Calculus *Springer, Berlin*, 1954.

[6] G. Soos, Uber die geodatischen Abbildungen von Riemannschen Raumen auf projektiv-symmetrische Riemannsche Räume. *Acta Mathematica Hungarica*, **9**(3-4): 359–361, 1958.

[7] K. Yano, On semi-symmetric connection. *Revue Roumaine de Math, Pure et Appliques*, **15**: 1570–1586, 1970.

[8] K. Yano, and M. Kon *Structures on manifolds*, world scientific publishing, 1989.

[9] K. Yano, Concircular geometry I. Concircular transformations. *Proceedings of the Imperial Academy*, **16**(6): 195–200, 1940.

[10] P. Zhao, and et al. Certain Curvature Conditions on Kenmotsu Manifolds Admitting a Quarter-Symmetric Metric Connection. *Publications de Linstitut Mathematique, Nouvelle srie 1*, **4**(118): 169–181, 2018.

[11] G. Zhen, and et al. On ξ-conformally flat contact metric manifolds. *Indian Journal of Pure and Applied Mathematics*, **28** (6): 725–734, 1997.

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