Research Paper

A COMPARISON OF SIX METHODS USED TO EVALUATE APPARENT THERMAL DIFFUSIVITY FOR SOILS (IĞDIR REGION, EASTERN TURKEY)

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ABSTRACT. The objective of this work is to investigate the influence of boundary conditions at depth soil on the development of methods to determine the soil s apparent thermal diffusivity based on solution of inverse problems of a heat-transfer equation. Experimental investigations were carried out to establish the influence of boundary conditions at depth in soil on the solution of inverse problems of modeling of heat transfer in soils. For this purpose, 1 soil profile in the land at different depths ($x = 0, 5, 10, 15, 20, 40, 60$ cm) thermal sensors (Temperature recorder Elitech RC-4) have been installed to measure soil temperatures depending on time and depths. Based on these data, the apparent thermal diffusivity in soils was calculated using the classical (layered) and proposed (point) methods developed for the case with one and two harmonics, and they were compared and the calculated characteristics were compared with the experimental results. It was found that the proposed point methods best reflect the movement of heat in the soil profile.

1. Introduction and Background

Soil thermal properties are important in agriculture and metrological applications. The soil thermal properties depend on its physical and hydro-mechanical properties, such as degree of saturation, dry density, solid fraction in the soil, organic matter content, the mineral composition of the soil, the water content in the soil, and the porosity filled with air [1, 3–5, 7, 13, 17, 20, 21]. Soil thermal properties such as heat capacity (C_v), apparent thermal diffusivity (κ), thermal conductivity (λ), damping depth (d), and heat absorptions (b), principally control soil temperature and soil heat flow. Determination of these parameters is important to understand behavior of soil thermal regime and manage soil temperature at field scale. Kim et al. (2018) reported that there are several in-situ methods for measuring thermal properties, including needle probes, divided bar, borehole relaxation, non-linear fitting, thermal gradients pulse, and estimation from thermal gradients. The generally accepted method for assessing the apparent thermal diffusivity of soil is the methods described in the works of Chudnovsky (1947), Carslaw, and Jaeger (1959) and et al [6–8, 10–19, 22]. These methods are based from observed soil temperature: on measuring the temperature of the day's surface and temperature at various depths of the soil. All of these researchers and others noted

FIGURE 1. Location of study area

that most of these models are based on solutions of the one-dimensional heat equation with constant diffusion. (Gao et al., 2009; Carslaw & Jaeger, 1959; Horton et al., 1983; Kaganov & Chudnovsky, 1953; Kolmogorov, 1950; Mikayilov & Shein, 2010; Nerpin & Chudnovskii, 1967; Tong et al., 2017; Wang et al., 2010). In modeling heat-transfer processes in soil, there is a need for analysis of solution of heat transfer equations, since for practical calculations of a thermal regime, use can be made of approximate solutions that have a simpler form and possess a sufficient accuracy, which must be,as far as possible, in fuller conformity with the physical picture of the processes of heat propagation in soil. To this end, it is expedient to make an analysis of the impact on heat transfer in soil using simplifications most frequently used in practice and to assess the influence of boundary conditions, individual terms, and coefficients of differential equations, and the effects of the dimensions of the region on the process being described. The accuracy and adequacy of physically substantiated models are determined first of all by experimental information on physical, hydrophysical, and physio-chemical properties of individual layers (horizons) of soils and their conditions on the upper and lower boundaries. The objective of this study was investigate the infl uence of boundary conditions in the soil depth under natural conditions on the development of methods to determine the soil s apparent thermal diffusivity based on solution of inverse problems of a heat-transfer equation. In this case, the thermal conductivity and diffusion coefficient were predicted on the undisturbed soil profile by four different classical and two improved methods, and the results were compared.

2. Materials and Methods

The research was carried out by opening a profile at 2 m length and 60 cm width in the land of Iğdır University Agricultural Application and Research Center located within the borders of Melekli Municipality of Iğdır Province (Figure 1). The geographical location of the study area is between 39,1°–39,5° East longitude and 44° -44,2° North latitude. Located at 850 m above sea level, the Igdir region covers approximately 3.588 (1.479) km^2 plain exhibiting a non-uniform characteristics in terms of topography. Thermal sensors are used for measuring and recording soil temperature in the soil profile with the depths of 0, 5, 10, 15, 20, 40 and 60 cm.

In this study, soil temperature was measured with a portable thermal Sensor (Elitech RC-4 Mini temperature data logger- See Figure 2). The sensor registers and stores temperature measurements in its memory. Recorded temperature degrees are stored directly and can be downloaded by users via data cable.

FIGURE 2. Placing the thermel sensors (Elitech RC-4 Mini temperature data logger) in soil profile

3. **Selection of a Model of Heat Transfer in Soil**

The one-dimensional distribution of the temperature field in an isotropic medium is described by the following classical equation of heat conduction [2–5]

$$(3.1) \qquad C_v(Z,t)\frac{\partial T(z,t)}{\partial t} = \frac{\partial}{\partial z}\left[\lambda(z,t)\frac{\partial T(z,t)}{\partial t}\right], \quad (z,t) \in (0,t) \times (0 < z < L)$$

where $T(z,t)$ is the soil temperature (K or °C) at depth $z(m)$, t is time (sec); C_v is the volumetric heat capacity of the soil (J.m^{-3}.K$^{-1)}$ and $C_v = \rho_b.C_m$, where ρ_b is the soil bulk density (kg · m^{-3}); C_m is the specific heat capacity (J.kg$^-$1.K); λ is the thermal conductivity (W.m^{-1}.K^{-1}), which can vary with soil moisture content. Assuming that a soil is vertically homogeneous, and that Cv and λ are independent of depth and time, Equation (3.1) becomes [1–22]

$$(3.2) \qquad\qquad\qquad \frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial z^2}$$

where $\kappa(= \lambda/C_v)$ is the apparent soil thermal diffusivity (m^2/s).

4. **Identification of Boundary Conditions of Heat Transfer in Soil**

The equation (3.1) must be solved analytically or numerically to find the temperature change at a given time t and at the depth z of soil. To solve equation (3.1), initial and boundary conditions are needed that reflect the natural process of heat transfer in the soil. The initial conditions correspond to the state of the variable at the zero (initial) time moment. For the theoretical description of the quasi-stationary regime problem (e.g., the daily or annual variation of the soil temperature), the initial condition is absent (so-called problems without an initial condition).

To study the influence of boundary conditions in modeling the transfer of heat in soil, we must set initial and boundary conditions. It is well known that the infl uence of the initial condition has no effect, in practice, on the distribution of the soil temperature at the instant of observation.

If the instant of time of interest is distant from the initial one, it makes sense to disregard the initial conditions, since their influence on the process becomes even weaker with time,

particularly, in the case of periodic formulation of the problem, e.g., of daily or annual variation in the soil temperature [2–4, 12–17]. The surface boundary condition of the first kind is written in the common form

$$(4.1) \qquad\qquad T(0, T) = \varphi(t)$$

where the function $\varphi(t)$ describes the changes in the soil surface temperature with time; the analytical form of the function should be determined in advance.

$$(4.2) \qquad\qquad T(0, t) = T_0 + \sum_{j=1}^{m} T_j \cos\left(j.\omega t + \epsilon_j\right),$$

where T_0 is the average daily (or annual) temperature of the active soil surface; T_j is the amplitude of the wave at the surface level for the jth harmonic; j is the index of the harmonic in the series; $\omega = 2\pi/\tau_0$ is the angular daily (or annual) frequency; τ_0 is the temperature wave period (days or years), for $\tau_0 = 24$ hours: $\omega = 7,2710^{-5} (rad/s)$; ϵ_j is the phase angle of the wave at the surface level for the j-th harmonic (radians); m is the harmonic number. The lower boundary condition is:

$$(4.3) \qquad\qquad T(x \to \infty, t) = T_0$$

or

$$(4.4) \qquad\qquad \partial T(x = L, t)/\partial x = 0$$

5. **Solution of the Direct Problem of Heat Transfer in Soil**

We should consider the solution of a problem in which the boundary conditions are specified and there are no initial conditions. The solution of (3.2) with boundary conditions (4.2) and (4.3) in dimensionless variables is of the form [2, 3].

$$(5.1) \qquad\qquad T(y, t) = T_0 + \sum_{j=1}^{m} T_j \exp\left(-b_j y\right) cos(j\bar{\omega}\tau + \alpha_j),$$

where

$$(5.2) \qquad y = \frac{z}{L}, \quad \tau = \frac{Kt}{L^2}, \quad \bar{\omega} = \frac{\omega L^2}{\kappa}, \quad b_j = L\sqrt{j\frac{\pi}{\tau_0 \kappa}}, \quad \alpha_j = \epsilon_j - b_j y$$

This problem was already studied by Fourier; it was first applied by Kelvin for the determination of the temperature variation in the soil of Edinburgh [3, pp. 78].

As correctly noted in [12, 13], when performing practical calculations, it is not possible to set the values of soil temperature at infinity as input data, since they are unknown. And it is impossible to measure it. Therefore, instead of $T(\infty, t)$, the temperature should be set at a certain depth L, starting from which for $x \geq L$ the quantity $T(x, t) = const$ or $\partial T(L, t)/\partial x = 0$. Thus, condition (4.4) more corresponds to the real conditions than boundary condition (4.3) [12, 13]. Therefore, one should also consider the boundary value problem (3.2), (4.2), and (4.2). The solution to this problem in dimensionless variables is as follows [12, 13].

$$(5.3) \qquad\qquad T(y, t) = T_0 + \sum_{j=1}^{m} \Phi_j(y, b_j). \cos\left[j\bar{\omega}\tau + \epsilon_j - \Psi_j(y, b_j)\right]$$

The functions Φ_j and Ψ_j are determined by:

$$(5.4) \qquad \Psi_j(b_j, y) = T_j \sqrt{\frac{\mathbf{ch(d_j)} + \cos{(\mathbf{d_j})}}{\mathbf{ch(2b_j)} + \cos{(\mathbf{2b_j})}}}, \quad \Phi_j(y, b_j) = \arctan\left[\frac{P_{2j}(y, b_j)}{P_{1j}(y, b_j)}\right]$$

where

$$(5.5) \qquad \begin{cases} P_{1j} = \mathbf{ch}(q_j)\cos{(b_j y)} + \mathbf{ch}(b_j y)\cos{(q_j)} \\ P_{2j} = \mathbf{sh}(q_j)\sin{(b_j y)} + \mathbf{sh}(b_j)sin(q_j) \end{cases} , \quad d_j = 2b_j(1-y), q_j = b_j(2-y)$$

6. **Solution of the Inverse Problem of Heat Transfer in Soil**

6.1. **Classical methods for determining the coefficient of apparent thermal diffusion.** On the basis of analytical solutions (5.1) of equation (3.2), various authors derived formulas for determining the apparent thermal diffusion coefficient [2–4, 8, 10, 13]. These methods are based on measuring the temperature of the day's surface and the temperature variation with depth soil. Below are the classic algorithms that are developed for the case when the daily variation of temperature on the soil surface is represented by one and two harmonics **1. Amplitude algorithm (Carslaw and Jaeger, 1959):**

$$(6.1) \qquad \kappa = \frac{\pi}{\tau_0} \cdot \frac{(z_2 - z_1)^2}{\ln^2\left[\frac{T_{\max}(z_1) - T_{\min}(z_1)}{T_{\max}(z_2) - T_{\min}(z_2)}\right]}$$

2. Phase algorithm (Nerpin and Chudnovskii, 1967):

$$(6.2) \qquad \kappa = \frac{\pi}{\tau_0}\left(\frac{z_1 - z_2}{\phi_2 - \phi_1}\right)^2$$

3. Arctangent algorithm (Kaganov and Chudnovsky, 1953):

$$(6.3) \qquad \kappa = \frac{\pi(z_2 - z_1)^2}{\tau_0.\arctan^2\left\{\frac{[T_1 z_1 - T_3 z_1][T_2 z_2 - T_4 z_2] - [T_2 z_1 - T_4 z_1][T_1 z_2 - T_3 z_2]}{[T_1 z_1 - T_3 z_1][T_1 z_2 - T_3 z_2] - [T_2 z_1 - T_4 z_1][T_2 z_2 - T_4 z_2]}\right\}}$$

4. Logarithmic algorithm (Kolmogorov, 1950):

$$(6.4) \qquad \kappa = \frac{4\pi(z_2 - z_1)^2}{\tau_0.\ln^2\left\{\frac{[T_1 z_1 - T_3 z_1]^2 + [T_2 z_1 - T_4 z_1]^2}{[T_1 z_2 - T_3 z_2]^2 + [T_2 z_2 - T_4 z_2]^2}\right\}}$$

where; τ_0 in the algorithms (6.1) period of heat wave, i.e. being the period of the fundamental cycle (24 h for daily temperature); $T_{min}(z)$ and $T_{max}(z)$ are the minimum and maximum temperature values during the measurement at the depths of $z = z_1$ and $z = z_2$ respectively; ϵ_1 and ϵ_2 in the algorithms (6.2) are the initial phases of the soil temperature at the depths of $z = z_1$ and $z = z_2$ respectively; $T_i(z_1)$ and $T_i(z_2)$ in the algorithms (6.3)-(6.4) are the soil temperature values at the depths $z = z_1$ and $z = z_2$, respectively, at the time moment $t_i = i\tau_0/4$ (i=1,2,3,4)(for our example $\tau_0 = 24$ h and $t_1 = 6$, $t_2 = 12$, $t_3 = 18$ and $t_4 = 24$ h)

6.2. **Improved methods for determining the coefficient of apparent thermal diffusion.** In contrast to the above algorithms, we continued to develop new methods that take into account the actual process of heat transfer in the soil. To do this, we used the solution(5.3) derived for a semi-bounded soil thickness [12, 13]. These suggested methods for calculating the apparent thermal diffusivity of soil k are presented below.

1. Suggested algorithm-1 (Mikayilov and Shein, 2010):

$$(6.5) \qquad \kappa_i = \frac{\pi}{\tau_0} \frac{2(z_i)^2}{\ln^2 \frac{[T(z_i,t_1)-T(z_i,t_3)]^2+[T(z_i,t_2)-T(z_i,t_4)]^2}{4T_a^2}}$$

2. Suggested algorithm-2 (Mikayilov and Shein, 2010):

$$(6.6) \qquad \kappa_i = \frac{\pi}{\tau_0} \cdot \left(\frac{L}{b_i^*}\right)^2$$

where; $T(z_i, t_j)$ in the algorithm (6.5) are the soil temperature values at the depth $z = z_i$, at the time at the time moment $t_j = j\tau_0/4$ (j=1,2,3,4) (for our example $\tau_0 = 24$h and $t_1 = 6$, $t_2 = 12$, $t_3 = 18$ and $t_4 = 24$h). Using solution (5.3) for one harmonic, i.e. $m = 1$, first, for an arbitrary dimensionless depth and time $t_j = j\tau_0/4$ $(j = 1,2,3,4)$, the following four equations are written:

$$(6.7) \qquad T(y, t_i) = T_0 + \Phi(y, b). \cos\left(i\frac{\pi}{2} + \alpha\right), \quad (i = \overline{1,4})$$

since it takes place $\bar{\omega}\tau_i = \frac{\omega L^2}{\kappa} \cdot \frac{\kappa}{L^2} t_i = \omega t_i = \frac{2\pi}{\tau_0} \cdot i \cdot \frac{\tau_0}{4} = \frac{\pi}{2} \cdot i$ and then we have: $\bar{\omega}\tau_i = \frac{\pi}{2} \cdot i$ After some transformations of equations (6.7) we have:

$$(6.8) \qquad \sum_{i=1}^{2} [T(y, t_i) - T(y, t_{i+2})]^2 = 4\Phi^2(y, b)$$

Taking into account the notation (5.4)-(**??**) for the function $\Phi_1(y, b_1)$ and $\Psi_1(y, b_1)$ in equality (6.8), we have the following expressions, which correspond to the boundary conditions (4.3) and (4.4):

$$(6.9) M_{\exp}(y, b) = \frac{\sum_{i=1}^{2} [T(y, t_i) - T(y, t_{i+2})]^2}{4T^2} = \frac{ch\,[2b(1-y)] + \cos\,[2b(1-y)]}{ch(2b) + \cos(2b)} = M_{cal}(y, b)$$

The definition of κ using formula (6.9) is carried out by the method of selecting on a computer the value of the parameter b_1^* from the condition of the coincidence of the value of the left side and the right side calculated from the initial data, that is, $M_{\exp}(y, b)$.

From the relationship $b^* = L\sqrt{\pi/\tau_0\kappa}$, we find the value of the apparent thermal diffusivity coefficient κ at a depth of $z = z_*$, which is equal to

$$(6.10) \qquad \kappa_i = \frac{\pi}{\tau_0} \cdot \left(\frac{L}{b_i^*}\right)^2,$$

where b* are the values of the parameter b, at which $|M_{exp} - M_{cal}| \leq 10^{-5}$

Depth (z) m	Clay	Slit %	Sand	BD kg m^{-3}	OM %	θ	C_v
0-10	28.60	32.10	39.80	0.9743	1.40	0.2406	1757.5904
20-25	25.50	29.50	45.10	1.0491	2.35	0.2584	1901.3979
30-35	14.20	19.60	66.30	1.1581	3.07	0.2672	2033.1646
50-60	22.45	33.35	44.20	1.3431	1.60	0.1870	1820.3163
0-60	22.69	28.64	48.85	1.1312	2.11	0.2383	1878.1173

TABLE 1. Some physical and chemical properties of research soil

Comparison of Methods The performance of four methods was evaluated by Pearson's Correlation Coefficient (r), Coefficient of Determination (R^2), Root Mean Squared Error (RMSE, σ), Mean Absolute Percentage Error (MAPE, A), and Theil's U Statistic (UII).

7. **Results and Discussion**

The main thermal characteristics of the soil are the coefficients of volumetric heat capacity (C_v), thermal conductivity (λ), thermal diffusivity (κ), thermal absorptive (assimilability), heat flow on the soil surface (Q) and the damping depth (d) of diurnal temperature waves, which are calculated by the following formulas [5, 20]

$$(7.1) \quad C_v = \rho_b.C_{m.s} + C_{v,w}.\theta = \rho_b \left[C_{m.org}.\frac{m_{org}}{m} + C_{m.min}.(1 - \frac{m_{org})}{m} \right] + \rho_w.C_w.\theta$$

$$(7.2) \qquad \lambda = \kappa.C_\nu, \qquad d = \sqrt{\tau_0 \kappa / \pi}, \qquad e = C_v \sqrt{\kappa} = \sqrt{\lambda C_v}$$

where, $C_{m,s}$ is the specific heat of the soil's solid part, $J/(kg°C)$; ρ_b is the soil bulk density, kg/m^3; $C_{v,w}$ is the volumetric heat capacity of the soil moisture equal to 4186.6 106 $kJ/(m^{3°}C)$; Cw is the specific heat of water, 4186.6 106 $kJ/(kg°0C)$; ρ_w is the water density, kg/m^3; θ is the volumetric moisture content (m^3/m^3); $C_{m,org}$ and $C_{m,min}$ are the specific heats of the organic and mineral components of the soil solid phase, respectively, $J/(kg°C)$; m_{org} is the mass of soil organic matter, kg; m is the soil mass (kg); m_{org}/m is the content of organic substance in soil. Vertical changes of soil properties in soil profile and soil column are depicted in Table 1. Figure 3 gives the daily behavior of surface and soil temperatures at different depths.

To determine the parameters of the soil s active surface (T_0, T_i and ϵ_i), we adopted one and two harmonics in condition (4.2). Using the measurement results for $x = 0$, that is, $T(x = 0, t_i)$ and using the least squares method, we determined the parameters of the temperature distribution of the surface of in studied soils.

Table 2 gives results of calculation of the parameters, and also statistical characteristics of approximation between the initial data and the data computed from (4.2) for $n = 1$ and $n = 2$. As can be seen from Table 2, the introduction of the second harmonic makes it possible to determine with high accuracy the parameters of the temperature distribution on the soil surface. Mean values for soil thermal diffusivity (κ), thermal conductivity (λ) and damping depth (d) calculated by amplitude, phase, arctangent and logarithm methods are given in Table 3. After determining the values of the thermal diffusivity κ, using the classical and improves formulas (6.1)-(6.2), the values of $T(z, t)$ for the depths of 5, 10, 15, 20, 40, and 60 cm
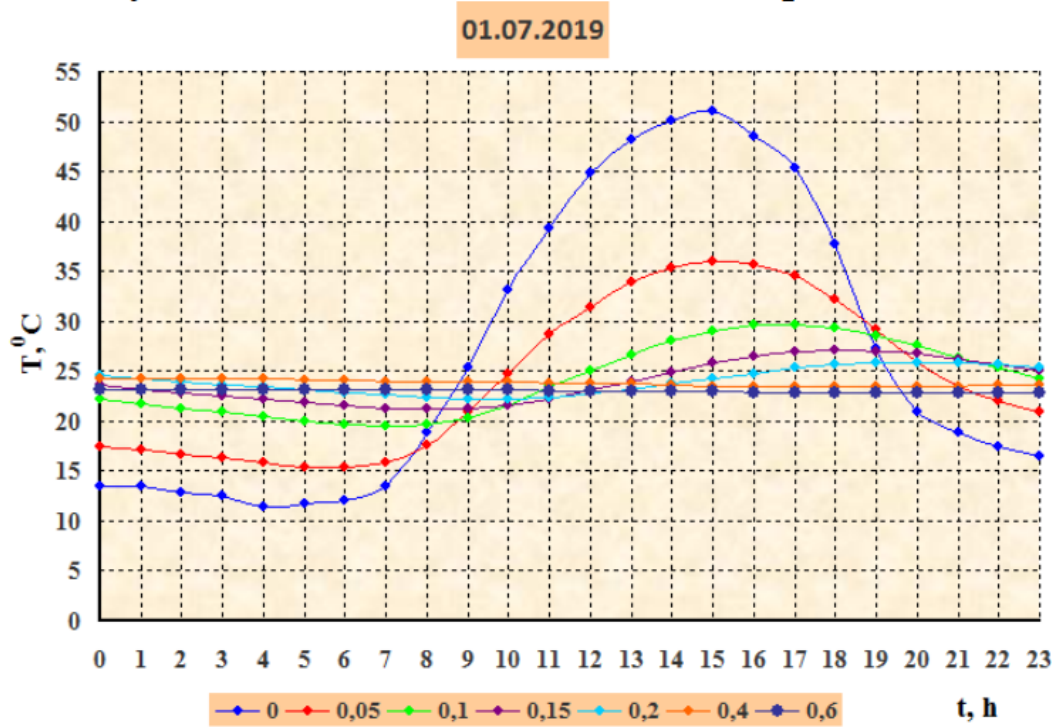
FIGURE 3. The hourly behavior of surface and soil temperatures at different depths

$\mathcal{N}_{\bar{0}}$	The parameters of soil surface	number of harmonics			
			m=1		m=2
1	Mean Temperature at soil surface $(T_0),^\circ C$	T_0	26.8488	T_0	26.8488
2	Amplitude of oscillationsof the soil surface temperature $(T_i),^\circ C$	T_1	19.5764	T_2	5.5732
3	Phase shift	ϵ_1	2.4929	ϵ_2	-1.0678
$\mathcal{N}_{\bar{0}}$	**Statistical approximation parameters**				
1	Coefficient of Determination, %	$\mathbb{R}^2$	91.77	$\mathbb{R}^2$	99.21
2	Root Mean Squared Error (RMSE) T in t	σ	4.54	σ	1.48
3	Mean Absolute Percentage Error (MAPE), %	A	18.73	A	3.91
4	Normalized Standard Error or Theil's U Statistic	UII	0.1359	UII	0.0421

TABLE 2. Parameters of the field soils surfaces and model performance

were calculated for the soil temperature. A comparison of numerical calculations revealed that improved methods 1 and 2, with a comparison with all classical methods, give satisfactory results (see Table 4)). Therefore, when modeling heat transfer in soils, it is necessary to take into account conditions of the second kind on the lower boundary. And to determine the parameter thermal diffusivity it is advisable to use Method 2.

$\mathcal{N}_{\bar{0}}$	Type of methods	C_v KJ/(m^3.°C)	$10^{-6}.\kappa$ m^2/s	λ W/(m.°C)	d m	e W.h$^{0.5}$/(m^2.°C)
			Classical Methods			
1	Amplitude	1878.1173	0.5416	1.0172	0.1220	23.0362
2	Phase	1878.1173	6.2780	11.7908	0.4155	78.4300
3	Arctangent	1878.1173	13.2986	24.9764	0.6048	114.1498
4	Logarithm	1878.1173	0.4889	0.9183	0.1160	21.8878
			Improved Methods			
5	Methods–5	1878.1173	0.2878	0.5405	0.0890	16.7917
6	Methods–6	1878.1173	0.2701	0.5074	0.0862	16.2693

TABLE 3. The mean values of the thermal diffusivity (κ), thermal conductivity (λ), damping depth (d) and heat absorptivity (e) of studies soils, calculated by six different methods.

z	σ				A			
	Numbering of methods				Numbering of methods			
m	1	4	5	6	1	4	5	6
0.05	3.876	3.836	3.940	1.560	12.534	12.196	11.876	3.841
0.10	3.927	3.780	3.448	2.068	12.301	11.661	10.980	4.645
0.15	3.728	3.610	3.408	2.603	11.727	11.717	12.284	6.175
0.20	3.364	3.279	3.168	2.457	11.667	11.721	12.006	5.745
0.40	3.151	3.160	3.158	1.640	12.586	12.657	12.664	3.739
0.60	3.992	3.993	3.993	2.219	16.596	16.596	16.597	5.302

TABLE 4. Values of parameters   and A for comparing methods 1–6. RMSE (σ)–Root Mean Squared Error, MAPE (A)–Mean Absolute Percentage Error.

8. CONCLUSIONS

When modeling heat transfer in soils, it is necessary to take into account conditions of the second kind on the lower boundary. And to determine the parameter thermal diffusivity, it is advisable to use Method 2. In fact, at certain depths, convective heat transfer is absent and the heat flux is zero, so it is necessary to accept the boundary conditions of the second kind. If the depth of profile is relatively small, then it is advisable to use Method 1, since this method also takes into account the values of the amplitude of the soil surface.

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